APPM 2360: Midterm exam 2
October 19, 2016

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your instructor's name, (3) your recitation section number and (4) a grading table. Text books, class notes, cell phones and calculators are NOT permitted. A one page (letter sized **1 side only**) crib sheet is allowed.

Problem 1: (24 points) **True/False** (answer True if it is always true; otherwise answer False) No justification is needed.

(a) The system of equations $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ has at least one solution $\vec{x}$ for any values of a, b, c, d, e, f.

(b) The polynomials $\{x^2+1, x+2, x^2-x-1\}$ form a basis for $\mathbb{P}_2$.

(c) If $\vec{v}_i$, $i = 1, 2, \ldots 5$ are vectors, and $\mathbb{W} = \text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5\}$, then the dimension of $\mathbb{W}$ is always 5.

(d) The set of all 4×4 matrices that have 0 entries on the diagonal is a vector space.

Problem 2: (24 points) **Short Answer** questions. A short justification is sufficient but *do not* simply answer True/False or Yes/No.

(a) Are the vectors $[0, 1, 1], [0, 2, 0], [2, 0, 1] \in \mathbb{M}_{13}$ (the space of 1×3 matrices) linearly independent or linearly dependent?

(b) Consider the set of all solutions to the differential equation

$$y'' + y = \sin(2t).$$

Is this set a vector space? (Remember to briefly justify your answer).

(c) Consider the set

$$W = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ k \end{bmatrix} \right\}.$$

(i) For what value(s) of k does W constitute a basis for $\mathbb{R}^3$?

(ii) If $\mathbb{V} = \text{Span}(W)$, for what value(s) of k is $\dim(\mathbb{V}) = 2$?

Problem 3: (26 points) Consider the matrix

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix}.$$

(a) Write down the characteristic polynomial that determines eigenvalues of A.

(b) Find the eigenvalues of A.

(c) Find the eigenvector(s) of A for its **largest** eigenvalue. (only the LARGEST λ!)

(d) What is the dimension of the eigenspace $\mathbb{E}_\lambda$ for the eigenvalue in (c)?

*****TEST CONTINUES ON OTHER SIDE OF PAGE*****

Problem 4: (24 points)

(a) Consider the following two matrices:

$$A = \begin{bmatrix} 1 & 4 & 7 & 9 \\ 0 & 2 & 11 & 12 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$

For each matrix, indicate 1) its rank and 2) whether its inverse exists. Do not calculate the inverse; only indicate whether or not it exists. In each case, explain your answer and/or show your work.

(b) Consider the following system of equations $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 2 & 6 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ -1 \\ k \end{bmatrix}$.

(i) For which value(s) of k is the system consistent?

(ii) For a value of k for which the system is consistent, determine at least **two** solutions.

Problem 5: (24 points) Consider the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 2 & 4 & 3 & 5 & 2 \end{bmatrix}$

(a) Find the RREF of A.

(b) Find a basis for

$$\mathbb{U} = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} \right\}.$$

(Notice these are the column vectors of the matrix A.)

(c) Are the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}$, and $\vec{v}_4 = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 5 \end{bmatrix}$ linearly independent?

Explain. (Hint: they are column vectors of A).

Problem 6: (28 points) Consider the linear system:

$$\begin{array}{rcrcrcr} 2x & + & y & & & = & 0 \\ 5x & + & 3y & & & = & 1 \\ & - & 2y & + & z & = & -4. \end{array}$$

(a) Write this system in matrix form as $A\vec{x} = \vec{b}$.

(b) Compute A^{-1}.

(Hint: you can check your answer by testing if $A^{-1}A = \mathbf{I}$.)

(c) Use your answer from part (b) to compute $\vec{x}$. Do not row-reduce to solve for $\vec{x}$.