APPM 2360 Spring 2017: Project 1
Credit Modeling
Due Thursday February 23 before 11:59 PM on D2L

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1 Background

If we deposit money in a savings account, take out a car loan, or maintain a balance on a credit card, then we face the implications of compound interest. For example, when a person obtains the card through a bank or credit union, they agree to reimburse the credit company each time the card is used. As the person uses the card, a balance accumulates in their account. At the end of the month, if the balance is not completely paid, then the credit company applies interest to the account. Interest continues to be applied each subsequent month the remaining balance on the card is nonzero.

1.1 Compounding Interest

Often, an interest is expressed as the annual rate. That is, the percent of the outstanding balance that is charged as interest over a year. However, how frequently the rate is applied to the current balance may vary. This frequency is how often the loan *compounds*. If the interest is compounded annually, the formula to calculate the amount of money owed after the first year, is

$$y(1) = (1 + r)y(0),$$

where $y(t)$ is the outstanding balance after t years, and r is the annual interest rate.

How would this change if, instead, the loan compounded semiannually? Then, half the interest rate would be applied to the loan value every 6 months.

$$y(.5) = \left(1 + \frac{r}{2}\right) y(0)$$

$$y(1) = \left(1 + \frac{r}{2}\right) y(.5) = \left(1 + \frac{r}{2}\right)\left(1 + \frac{r}{2}\right) y(0) = \left(1 + \frac{r}{2}\right)^2 y(0)$$

This pattern continues for any frequency of compounding. That is, if a loan is compounded n times per year, the value of the loan is

$$y(1) = \left(1 + \frac{r}{n}\right)^n y(0). \tag{1}$$

The more frequently that interest compounds, the higher the value at the end of the year. However, there is a limit as n goes to infinity. The limit, which we call *continuously compounding* is:

$$y(1) = y(0)e^r$$

. This model can also be expressed as a differential equation:

$$y'(t) = ry(t), \quad y(0) = y_0. \tag{2}$$

Now, if one makes constant payments, P, on the credit card debt, the model differential equation then becomes

$$y'(t) = ry(t) - P, \quad y(0) = y_0. \tag{3}$$

2 The problem

Your roommates just opened up a credit card. Their money management skills are lacking and you are worried that they will forget monthly payments and be burdened by interest payments for years to come. However, despite your roommates' unfortunate inability to manage money they have a concrete understanding of differential equations. Your goal is to communicate the necessity of avoiding interest payments using your skills as a student in APPM 2360. Your friends have various options when choosing the credit card that may be best for their needs. The following points should help you know what to include in your analysis. In the following you will use the analytical tools you have acquired in APPM 2360 to discuss solutions to the model equation. Additionally, you will utilize Euler's (pronounced "oil-ers") method to approximate solutions to equation (2) and (3). For this project, you will write your own function implementing the explicit Euler method:

$$y_{n+1} = y_n + hf(t_n,\ y_n) \tag{4}$$

where y_n is an approximate solution to the ODEs at time t_n. That is, $y_n \approx y(t_n)$.

2.1 Fixed rate credit

We begin by assuming that the credit rate is fixed, i.e. r is a constant value.

- Assuming that $r = 0.14$, and the original balance is \$10,000, compute the total cost of the credit card debt after 5 years, compounded $1, 2, 4$, and 12 times per year, without any payments. On the same graph, plot (1) for $n = 1, 2, 4, 12$. Label each curve.
- Determine if (2) is a reasonable model for (1) when interest is compounded daily, n = 365.
- Are there any equilibrium solutions to (2)? If so, what are they? What about to (3)? What do these equilibria represent, in real-word terms?
- Plot the direction field for (3) with $r = 0.14$, $P = 750 * 12 = 9000$. What does $\frac{dy}{dt} < 0$ represent? What about $\frac{dy}{dt} > 0$?
- Solve (3) using separation of variables with leaving y_0, r and P general.
- Using your solution to the previous problem, find a value for P so that the debt is paid within T years. Explain the usefulness of this formula. How will this prove useful to your roommate?

2.2 Programming Euler's method

The differential equations we wish to solve often will be difficult to solve by hand so we enlist the help of a numerical scheme. Here we will utilize MATLAB to perform Euler's method.

- First we want to create each piece we need to calculate $y'(t)$. Create functions $r(t)$ and $p(t)$ that take t as an input and return the interest rate or payment rate respectively. For now we will use a constant interest rate of 14%, and monthly payment \$750. (However, you should make functions $r(t)$ and $p(t)$ because later we will consider credit cards with non-fixed interest rates $r(t)$. These cards may have a rate increase after an introductory period as you will see below. You may also have a variable interest rate credit card that is tied to the "Fed Funds Target Rate" set by the Federal Reserve.)

- Now, we put the pieces together. Create a function y'(t,y), which takes in t and y as input arguments, and outputs the value of y' at that point. Note that we can put the functions r and p inside the function y'. At this step you are creating the right hand side of Euler's method.
- Finally, we must program Euler's method. Use a step size of .01, and use a *While* loop to run the method until the credit card debt is paid off ($y = 0$). When will the initial debt of $10,000 be paid off, with a constant interest rate of 14% and a monthly payment $750? Make sure to include a plot of $y(t)$ with all axes labeled and compare this to the analytical solution of the IVP. Include code you write in an appendix.

2.3 Analysis using non-fixed interest rates

Now we turn to credit cards with changing interest rates. Use the Euler's method code for these problems. You may also find the *find* command helpful. Type 'help find' into the command window to learn more. Suppose that the bank offers a 0% APR (annual percentage rate) for 12 months after debt is accumulated. After this, the rate is 18.5%.

- Suppose that your roommate pays $500 per month. How long will it take him/her to pay off a credit card debt of $10,000 using the adjustable APR mentioned above?
- How long will it take if they pay $750 per month?
- How much interest is paid in each case?
- Plot both scenarios on the same graph. Explain what is going on in it. How does the interest rate affect the graph?

2.4 Analysis using variable monthly payments

Suppose that the bank offers a constant interest rate of the rate at 18.5%. Your friends tell you that they are anticipating steady salary increases over the next few years. Suppose that they make credit card payments at a monthly rate of $800 + 10t$, where t is the number of months since the loan was made.

- Assuming that this payment schedule can be maintained, when will the loan be fully paid?
- Assuming the same payment schedule, how large a loan could be paid off in exactly 20 years?

2.5 A message to your roommate

Based on your knowledge gained previously, write a message to your roommate communicating your findings regarding credit card debt in this project. Your writing should be clear and concise while still providing enough detail that your roommate will believe your findings are correct.

3 Report Guidelines

You and your group will submit your project on D2L, in the appropriate dropbox (you can find these under the "assessments" tab in D2L) Your group must:

- You must work in a group of three (you cannot work alone). Working in larger or smaller groups will result in a significant penalty for all group members.

- Do not put off finding a group, do this early. You should have a group set up within one week of the project assignment date.
- If you cannot find a group, e-mail the instructors. For this project groups, e-mail *igor.rumanov@colorado.edu* (Igor Rumanov - coordinating instructor).
- Submit your project in pdf format. When word documents are uploaded to D2L, the equations in them are commonly jumbled around.
- Submit code used for your project (.nb file(s) for Mathematica, .m file(s) for MatLab, etc).
- Have only ONE group member submit the project. Having multiple people in your group submit the project to D2L will result in multiple grades, and we will take the LOWEST one.
- Include the names of all group members working on the project.

Your report needs to accurately and consistently describe the steps you took in answering the questions asked. This report should have the look and feel of a technical paper. Presentation and clarity are very important. Here are some important items to remember:

- Absolutely make sure your recitation number is on your submitted report.
- Start with an introduction that describes what you will discuss in the body of your document. A brief summary of important concepts that you will be using in your discussion could be useful here as well.
- Summarize what you have accomplished in a conclusion. No new information or new results should appear in your conclusion. You should only review the highlights of what you wrote about in the body of the report.
- Always include units in your answers.
- Always label plots and refer to them in the text. The main body of your paper should NOT include lengthy calculations. These should be included in an appendix, and referred to in the main body.
- Labs must be typed. Including the equations in the main body (part of your learning experience is to learn how to use an equation editor). An exception can be made for lengthy calculations in the appendix, which can be hand written (as long as they are neat and clear), and minor labels on plots, arrows in the text and a few subscripts.
- Your report doesn't have to be long. You need quality, not quantity of work. Of course you cannot omit any important piece of information, but you need not add any extras.
- DO NOT include printouts of computer software screens. This will be considered as garbage. You simply need to state which software you used in each step, and what it did for you.
- You must include any plot that supports your conclusions or gives you insight in your investigations.
- Write your report in an organized and logical fashion. Section headers such as Introduction, Background, Problem Statement, Calculations, Results, Conclusion, Appendix, etc... are not mandatory, but are highly recommended. They not only help you write your report, but help the reader navigate through your paper, besides giving it a clearer look.