APPM 2360: Midterm exam 1
September 21, 2016

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your instructor's name, (3) your recitation section number and (4) a grading table. Text books, class notes, cell phones and calculators are NOT permitted. A one page (letter sized **1 side only**) crib sheet is allowed.

Problem 1: (36 points, 6 points each) **True/False** (answer True if it is always true otherwise answer False) or **Short Answer** for the following problems. No justification is needed.

(a) The differential equation $y' + y\sin^2 t = t^2 y + 1 - t^2 - y\cos^2 t$ is separable. (True/False)

(b) Picard's Theorem tells us that the IVP $y' = t\sqrt{y}$, $y(1) = 0$ has a unique solution. (True/False)

(c) Consider the logistic equation $p' = 2(1 - \frac{p}{100})p$. If $p_1(t)$ and $p_2(t)$ are both solutions to the equation, then $p(t) = p_1(t) + p_2(t)$ is always a solution. (True/False)

(d) Given the fact that $y(t) = e^{2t}$ is a solution to the differential equation

$$y'(t) + p(t)y(t) = 2e^{2t} + e^{3t}.$$

Find the function $p(t)$. (Short Answer)

(e) Consider the coupled system of equations

$$\frac{dx}{dt} = 9x - 3xy$$
$$\frac{dy}{dt} = -2y + xy.$$

Find the vertical nullcline(s) of this system. (Short Answer)

(f) For the following differential equation

$$\frac{dy}{dt} = y(3 - y),$$

find all equilibrium solutions and classify them as stable, unstable or semistable. (Short Answer)

Problem 2: (30 points) Consider the differential equation

$$\frac{dy}{dt} = -2y\sin t - 2\sin t \tag{1}$$

(a) Find the general solution to Eq. (1) using separation of variables.

(b) Demonstrate that your solution from (a) indeed satisfies the differential equation (1).

(c) Find the unique solution to Eq. (1) that passes through $(t = \pi/2, y = 5)$

(d) What is the nature of the solution that passes through $(t = 0, y = -1)$?

TEST CONTINUES ON OTHER SIDE OF PAGE

Problem 3: (30 points) Consider the initial value problem:

$$ty' + (t^2 + 1)y = te^{-t^2}, \quad y(2) = 0.$$

(a) Find the solution to the homogeneous equation.
(b) Using the variation of parameters method, find a particular solution.
(c) Determine the general solution to the differential equation.
(d) Determine the solution to the initial value problem.

Problem 4: (30 points) [Note: if your answer involves logarithms, you may leave these unevaluated]

(a) A scientist begins an experiment several years ago starting with 32/9 grams of a radioactive substance. Last year, only 2 grams of the substance remained, and this year (exactly 1 year later), only 1.5 gram of the substance remain. How many years ago did the scientist begin the experiment?
(b) What is the half-life of the radioactive substance?

Problem 5: (24 points) Suppose that a tank contains 100 gallons of water with an initial salt concentration of 5 oz/gal. A solution with a concentration of 10 oz/gal of salt is added at a rate of 5 gal/min and the well-stirred mixture drains from the tank at the same rate.

(a) Set up an initial-value problem describing the amount of salt in the tank after t minutes.
(b) Find the solution to this IVP.
(c) What is the long-term behavior of this solution?