MR4850537 53A04 35K15 53E99

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Evolving an immersed star-shaped curve to another one with same winding number. (English. English summary)

J. Differential Equations **424** (2025), 421–437.

This paper addresses a variant of Yau's problem by introducing a parabolic curve flow that evolves one immersed star-shaped curve into another star-shaped curve with the same winding number in the plane, provided both curves are star-shaped with respect to the origin and share an identical geometric quantity $E = \int \frac{d\theta}{r(\theta)}$. The flow is constructed using a nonlocal term that preserves this quantity, and through reduction to a semilinear parabolic equation for the radial function, the authors establish short-time existence, prove that the evolving curve remains star-shaped for all time, and demonstrate global existence of the flow. By applying Harnack-type estimates and uniform gradient bounds, they show that the radial function converges smoothly to that of the target curve as time tends to infinity, which implies convergence of the evolving curve itself.

The main contribution lies in extending Yau's problem from convex curves to the broader class of immersed star-shaped curves, with Fourier analysis revealing exponential decay of higher modes and numerical experiments validating the theoretical results, thereby laying a foundation for possible higher-dimensional generalizations.

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