MR4477057 53E10 52A39

Wei, Yong (PRC-HEF-SM); **Yang, Bo** (PRC-ASBJ-AMS)

Volume preserving flows for convex curves and surfaces in the hyperbolic space. (English. English summary)

J. Funct. Anal. **283** (2022), *no.* 11, *Paper No.* 109685, 28 *pp.*

Let M be a smooth closed curve ($n=1$) or hypersurface ($n\geq 2$) in the hyperbolic space $\mathbb{H}^{n+1}$, with principal curvatures denoted by $\kappa_1:=\kappa,\kappa_2,\dots,\kappa_n$. There are three convexity conditions for M. It is said to be

- convex, if $\kappa_i>0$ for all $i=1,\dots,n$;
- h-convex, if $\kappa_i\geq 1$ for all $i=1,\dots,n$;
- of positive sectional curvature ($n\geq 2$), if $\kappa_i\kappa_j>1$ for all integers $1\leq i\neq j\leq n$.

The authors first study κ^α type area-preserving and length-preserving curvature flows of smooth closed convex curves in the hyperbolic plane $\mathbb{H}^2$. Motivated by previous work on curve flows [D.-H. Tsai and X.-L. Wang, Calc. Var. Partial Differential Equations **54** (2015), no. 4, 3603–3622; MR3426088], they show that these two types of flows exist globally and evolve convex curves to geodesic circles exponentially in C^∞ topology. This theorem strengthens a part (the $n=1$ case) of Theorem 1.2 in [B. Andrews and Y. Wei, Geom. Funct. Anal. **28** (2018), no. 5, 1183–1208; MR3856791], where the initial curve is assumed to be h-convex.

The authors also consider the volume-preserving Gauss curvature flow of smooth closed convex surfaces in $\mathbb{H}^3$. It is proved that the evolving surface remains convex and smoothly converges to a geodesic sphere exponentially as $t\to\infty$. This result is a generalization of the $n=2$ case of Theorem 1.2 in [B. Andrews, X. Z. Chen and Y. Wei, J. Eur. Math. Soc. (JEMS) **23** (2021), no. 7, 2467–2509; MR4269419], where the initial surface is of positive sectional curvature.

In the proofs of both above-mentioned results, a technique by K.-S. Chou [Comm. Pure Appl. Math. **38** (1985), no. 6, 867–882; MR0812353] plays an important role in obtaining uniform bounds on curvatures. *Laiyuan Gao*

[References]

1. B. Andrews, Evolving convex curves, Calc. Var. Partial Differ. Equ. 7 (1998) 315–371. MR1660843
2. B. Andrews, X. Chen, Y. Wei, Volume preserving flow and Alexandrov-Fenchel type inequalities in hyperbolic space, J. Eur. Math. Soc. 23 (2021) 2467–2509. MR4269419
3. B. Andrews, Y. Wei, Quermassintegral preserving curvature flow in hyperbolic space, Geom. Funct. Anal. 28 (5) (2018) 1183–1208. MR3856791
4. M.C. Bertini, G. Pipoli, Volume preserving non-homogeneous mean curvature flow in hyperbolic space, Differ. Geom. Appl. 54 (2017) 448–463. MR3693942
5. A. Borisenko, V. Miquel, Total curvature of convex hypersurfaces in the hyperbolic space, Ill. J. Math. 43 (1) (1999) 61–78. MR1665641
6. E. Cabezas-Rivas, V. Miquel, Volume preserving mean curvature flow in the hyperbolic space, Indiana Univ. Math. J. 56 (2007) 2061–2086. MR2359723
7. J. Escher, G. Simonett, The volume preserving mean curvature flow near spheres, Proc. Am. Math. Soc. 126 (9) (1998) 2789–2796. MR1485470
8. M.E. Gage, On an area–preserving evolution equation for plane curves, in: Nonlinear Problems in Geometry, in: Contemp. Math., vol. 51, Amer. Math. Soc., 1985, pp. 51–62. MR0848933
9. L. Gao, S. Pan, D.-H. Tsai, On a length-preserving inverse curvature flow of convex closed plane curves, J. Differ. Equ. 269 (7) (2020) 5802–5831. MR4104943

10. L. Gao, S. Pan, D.-H. Tsai, On an area-preserving inverse curvature flow of convex closed plane curves, J. Funct. Anal. 280 (8) (2021) 108931. MR4212066
11. C. Gerhardt, Curvature Problems, Series in Geometry and Topology, vol. 39, International Press, Somerville, MA, 2006. MR2284727
12. M.A. Grayson, Shortening embedded curves, Ann. Math. 129 (1989) 71–111. MR0979601
13. P. Guan, J. Li, A mean curvature type flow in space forms, Int. Math. Res. Not. 2015 (13) (2015) 4716–4740. MR3439091
14. R. Hamilton, Four-manifolds with positive curvature operator, J. Differ. Geom. 24 (2) (1986) 153–179. MR0862046
15. Gary M. Lieberman, Second Order Parabolic Differential Equations, World Scientific, 1996. MR1465184
16. L. Ma, A.Q. Zhu, On a length preserving curve flow, Monatshefte Math. 165 (2012) 57–78. MR2886123
17. M. Makowski, Mixed volume preserving curvature flows in hyperbolic space, arXiv:1208.1898. MR3016508
18. S. Montiel, A. Ros, Compact hypersurfaces: the Alexandrov theorem for higher order mean curvatures, Pitman Monogr. Surv. Pure Appl. Math. 52 (1991) 279–296. MR1173047
19. L.A. Santaló, Integral Geometry and Geometric Probability, second edition, Cambridge University Press, 2004, with a foreword by Mark Kac. MR2162874
20. N. Sesum, D.-H. Tsai, X.-L. Wang, Evolution of locally convex closed curves in the area-preserving and length-preserving curvature flows, Commun. Anal. Geom. 28 (8) (2020) 1863–1894. MR4201791
21. D.-H. Tsai, X.L. Wang, On length-preserving and area-preserving nonlocal flow of convex closed plane curves, Calc. Var. Partial Differ. Equ. 54 (2015) 3603–3622. MR3426088
22. K. Tso, Deforming a hypersurface by its Gauss-Kronecker curvature, Commun. Pure Appl. Math. 38 (1985) 867–882. MR0812353
23. G. Wang, C. Xia, Isoperimetric type problems and Alexandrov-Fenchel type inequalities in the hyperbolic space, Adv. Math. 259 (2014) 532–556. MR3197666

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.