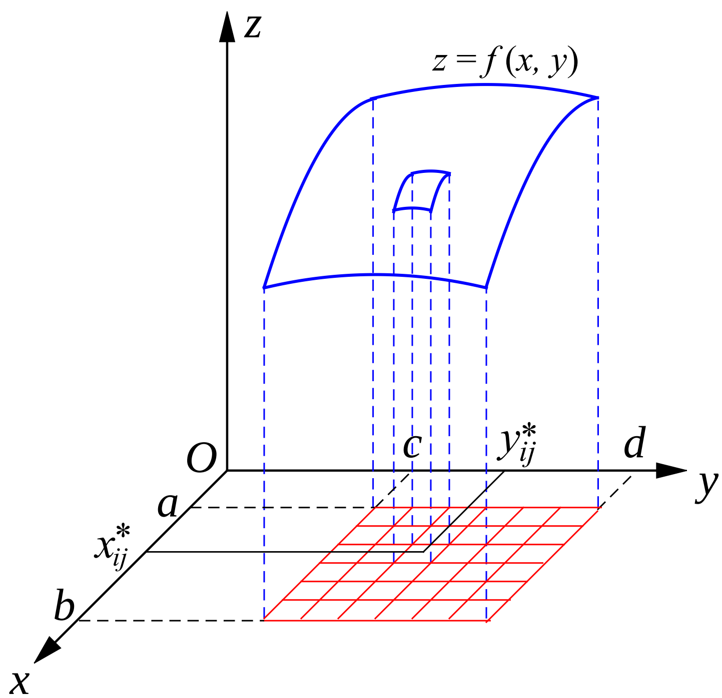
A Concise Course of Calculus



by

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Preface

Isaac Newton in 1675 said,“If I have seen further, it is by standing on the shoulders of Giants.” As one of the greatest achievements of the human intellect, Calculus is a giant in modern Mathematics. As you open this book, in front of you stands this giant. The main purpose of our composition of this book is to bring a guide for the readers who want to stand on the shoulders of this giant.

Calculus literally means (a method of) computation. Functions and the limits play central roles in this subject, so almost all computations in this book are about different kinds of limits for functions. The book begins with an introduction of elementary functions and different kinds of limits. The theory of differentiation and the applications are discussed in Chapter 2 and in Chapter 3, respectively. Chapter 4 is about the theory of integration, including definite and indefinite integrals. In Chapter 5, the Fundamental Theorem of Calculus is proved. Improper integrals, series of numbers and series of functions are also introduced in this chapter. In Chapter 6, multivariable calculus are established. The definite integral for functions with single variable is generalized to double and triple integrals for multivariable functions. In the last chapter, the definite integral on the intervals are further generalized to integrals on curves and surfaces, and some important integral formulas including Green’s, Gauss’ and Stokes’ are introduced. Especially, we mention Riemann’s idea of one and two dimensional manifolds and the Stokes Formula for differential forms as the end of this book.

In the journey to the giant’s shoulders, the readers will learn different kinds of techniques, such as the calculation of limits, the method of finding the maximum or minimum values of functions, the computation of derivatives and various integrals and so on. In order to help you master these skills, examples are presented after the introduction of each method. As an old saying goes, ”Practice makes perfect!” I strongly recommend the readers to compute every step in the examples by yourselves and solve all the problems in the exercise at the end of each section. I do not believe there is another way to handle Calculus without persistent hard work.

The Chapter 1 to the Chapter 5 are written by Laiyuan Gao. Chapter 6 and Sections 1 to 3 in Chapter 7 are finished by Shuixia Hao and Yuntao Zhang. The last two sections in Chapter 7 are completed by Laiyuan Gao. All the figures in this book are plotted by Laiyuan Gao.

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