

Basic Theory of the Curve Shortening Flow

by

Laiyuan Gao Shengliang Pan

Contents

Chapter 1 Introduction.....	1
1.1 Variation formulae.....	1
1.2 Existence of the solution.....	8
1.3 Basic properties.....	20
1.4 An outline of this book.....	28
Chapter 2 Gage-Hamilton Theorem.....	34
2.1 Gage's inequality.....	34
2.2 Short-time existence.....	40
2.3 Long term existence.....	42
2.4 Asymptotic behavior.....	49
Chapter 3 Singularity Analysis.....	60
3.1 Self-similar solutions.....	60
3.2 Type I singularity.....	69
3.3 Type II singularity.....	73
Chapter 4 Grayson Theorem.....	79
4.1 Grayson's original proof.....	79
4.1.1 The δ -whisker lemma.....	79
4.1.2 A sketch of Grayson's proof.....	87
4.2 Hamilton's proof.....	88
4.3 Huisken's proof.....	97
4.4 A proof by Andrews-Brayn.....	101

Chapter 5 Shortening Closed Curves.....	109
5.1 Closed curves along the CSF.....	109
5.2 Shortening Grayson's figure 8	116
Chapter 6 Chou-Zhu Theorem.....	121
6.1 Shortening open curves.....	121
6.2 Curves with finite total absolute curvature.....	121
6.3 Asymptotic behavior	125
Chapter 7 A survey of the CSF in Other Spaces.....	132
7.1 The CSF in \mathbb{E}^3	132
7.2 The CSF on complete surfaces.....	134
7.3 The CSF in compact manifolds	136
7.4 Some open problems	141
Bibliography	143
Appendix A Differential Geometry in \mathbb{E}^3	147